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Using High-Probability Instructional Sequences and Explicit Instruction to Teach Multiplication Facts

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Abstract

Students with learning disabilities often struggle with math fact fluency and require specialized interventions to recall basic facts. Deficits in math fact fluency can result in later difficulties when learning higher-level mathematical computation, concepts, and problem solving. The response-to-intervention (RTI) and multitiered-systems-of-support (MTSS) approaches for delivering research-based interventions to struggling learners provide educators with the structural frameworks necessary for planning tiered interventions to address skill deficits. Some schools have been implementing RTI/MTSS for years, while others have recently started using these frameworks. Regardless of the number of years delivering tiered interventions, educators benefit from learning about additional interventions they can implement for students requiring tertiary supports (i.e., Tier 3). This article provides readers with a detailed explanation of a Tier 3 multiplication fact fluency intervention that involves the use of high-probability instructional sequences and explicit, systematic, intensive instruction to increase motivation and fluency development.

Keywords

multiplication, behavioral momentum, explicit instruction, systematic instruction, intensive instruction, high-probability instructional sequences, response to intervention, multitiered systems of support, Tier 3, learning disabilities

Justin is a fourth-grade student who is currently being evaluated to determine if he is eligible for special education services due to his poor academic performance in mathematics. One of Justin's main struggles is with the rote memorization of multiplication facts. Although he has a strong conceptual understanding of multiplication and has committed many multiplication facts to memory since the time they were introduced in third grade, he does not fluently recall all multiplication facts. This puts him at a disadvantage when he is presented with mathematics lessons in the general education classroom that require the mastery of multiplication facts as a prerequisite to learning the targeted skills and concepts. His teacher had been providing small-group instruction to focus on multiplication fact fluency for some students in the classroom, including Justin, for 10 to 15 min two to three times each week. In the small group, the teacher used flash cards and multiplication fact worksheets, but Justin had been making minimal progress. According to his teacher, Justin has difficulty retaining new information in all subject areas, indicating an overall need for interventions that focus on addressing his ability to memorize and retain information. Thus, Justin received intensive Tier 3 interventions as part of his school's response-to-intervention (RTI) efforts to provide the explicit instruction he needs to learn his multiplication facts and commit them to memory (see Note 1).

Many students in classrooms across the country experience the same difficulties with math fact fluency that Justin faces on a daily basis. According to the National Mathematics Advisory Panel (NMAP) Final Report published by the U.S. Department of Education (NMAP, 2008), many students in the United States are not efficient in solving single-digit addition, subtraction, multiplication, and division with whole numbers and become overreliant on calculators for basic computation. Students who are not fluent with basic math facts struggle with mathematical problem solving and

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more advanced mathematical computation, resulting in low mathematical performance throughout their years in school (Skinner & Daly, 2010; Woodward, 2006). These mathematical deficits may also impact students later in life when they are pursuing careers that necessitate the mastery of basic math facts to perform job requirements.

Challenges students with learning disabilities (LD) face in learning basic math facts can be overcome when explicit instruction approaches are utilized (Kroesbergen & Van Luit, 2003). The RTI and multitiered-systems-of-support (MTSS) frameworks provide an avenue in which students with and without LD who struggle academically and/or behaviorally receive specialized instruction through the implementation of evidence-based tiered interventions and ongoing progress monitoring (L. S. Fuchs & Vaughn, 2012). However, classroom teachers may not have experience using a variety of interventions to solve specific learning challenges of students (Prasse et al., 2012).

This article provides readers with step-by-step procedures for implementing a tertiary (i.e., Tier 3) intervention for improving math fact fluency. The intervention consists of a combination of high-probability instructional sequences and explicit, systematic, intensive instruction. The purpose of the intervention is to effectively support students who struggle with learning multiplication facts by increasing their motivation to work on learning their facts (Rave & Golightly, 2014) and by using instructional strategies that improve their memorization of the learned facts. The high-probability instructional sequence improves motivation because students experience independent success easily and repeatedly (Banda, Matuszny, & Therrien, 2009). The explicit, systematic, intensive instruction provided using the A-B-C teaching sequence ensures immediate student success through the use of prompts and provides the repetition necessary to promote memorization (Leaf et al., 2013). Justin's response to this intervention will be shared to fully demonstrate how this intervention is implemented and how progress is monitored.

High-Probability Instructional Sequences and Explicit Instruction

The intervention uses a combination of high-probability instructional sequences and explicit, systematic, intensive instruction to improve multiplication fact fluency. The strategy of using high-probability instructional sequences (Mace et al., 1988) is founded on the behavioral momentum principle. *Behavioral momentum* refers to the tendency for behavior to continue following a change in environmental conditions (Nevin, Mandell, & Atak, 1983). Specifically related to the learning of multiplication facts, the high-probability instructional sequences strategy involves presenting students with a series of facts that can be quickly recalled by the student, or high-probability instructional sequences, prior to presenting a fact that the student cannot recall (i.e., low-probability request), thus building behavioral momentum or motivation to continue to engage and respond. Other terms used to describe this teaching approach include *interspersal procedures* and *high preference* procedures.

The use of high-probability instructional sequences alone is not sufficient for teaching multiplication fact fluency. In addition to improving student success and motivation to engage in the study of multiplication facts, students also need to learn how to commit unknown facts to memory. Thus, explicit, systematic, intensive instruction is paired with high-probability instructional sequences to teach the unknown facts when they are presented after a series of known facts.

The intervention is explicit because it is a direct teaching approach that includes unambiguous instructional procedures (Archer & Hughes, 2011) and provides modeling, multiple practice opportunities, and frequent student responses and immediate feedback (Swanson, 2001). It is systematic in that it is a replicable process (Snell, 1983) that promotes the mastery of new skills through careful design and instructional delivery (Ehlhardt et al., 2008) using data to show a functional relationship between the intervention and acquisition of the targeted skills (Spooner, Knight, Browder, & Smith, 2012). The intervention is considered intensive because it is delivered using 1:1 instruction on a daily basis for an extended period of time until mastery is achieved (D. Fuchs, Fuchs, & Vaughn, 2014).

There are many evidence-based approaches that can be utilized to deliver systematic instruction. This intervention focuses on the use of the A-B-C teaching sequence consisting of the following instructional procedures.

- Provide an antecedent or cue that evokes a response from the student (e.g., hold up a flash card).
- Deliver a prompt if the student is unable to respond to the antecedent independently (e.g., "3 × 4 = 12").
- The student demonstrates the expected behavioral response (e.g., supplies the correct answer to the fact on the flash card).
- Deliver a consequence to positively reinforce the student for demonstrating the expected behavior (e.g., praise).
- Pause briefly prior to delivering the cue for the next trial (Smith, 2001).

This format uses a quick pace of instruction, repetition of trials that provides increased learning opportunities, and instruction that occurs in the context of a structured teaching session (Duker, Didden, & Sigafoos, 2004). In this intervention, the A-B-C teaching sequence is used when an unknown fact is presented immediately following a high-probability instructional sequence. See Figure 1 for an explanation of the A-B-C teaching sequence procedures.

1. **Antecedent (A):** Hold up a flashcard of an unknown fact.
2. **Prompt:** Since the fact is unknown, prompt the student by providing the answer (e.g., "4 x 6 = 24"). Then immediately ask the student to supply the answer to the fact (e.g., "What is 4 x 6?"), allowing the student an opportunity for a successful response immediately following the prompt.
3. **Behavior:** The student states the correct answer to the fact.
4. **Consequence:** Provide positive reinforcement immediately following the correct response.
5. **Pause:** Pause briefly (2–3 seconds), then deliver the next trial following the same steps.
6. **Provide Repetition:** To support memorization, the same fact should be presented for three consecutive trials prior to starting the next high-probability instructional sequence (two or more known facts).

Figure 1. A-B-C teaching sequence procedures.

Table 1. Instructions for Gathering Baseline and Progress-Monitoring Data.

Step	Instruction
1	Give the student the following instructions: "I am going to hold up multiplication facts one at a time. You will look at the card, read it to yourself or read it aloud, and say the answer. If you do not know the answer, say, 'Pass.' I am just figuring out which facts you know right away. You do not need to try and figure out the answer by counting in your head or using your fingers." Do a practice trial to make sure the student knows to look at the card and either give the answer or say, "Pass." Note: If the student requires auditory input, you can alter the instructions so that you read the fact to the student before the student supplies the answer.
2	Shuffle the cards so that they are in random order.
3	Hold up the flash cards one at a time.
4	If the student supplies the answer in 3 s or less, put that card in a pile of known facts (these will be the facts used for high-probability instructional sequences). Remember that the purpose of this intervention is to build fluency. If a student counts to get the correct answer, the student is not fluent with that fact. In such cases, do not count that as a known fact, even if the student can count and get the correct answer in 3 s or less.
5	If the student says, "Pass"; answers incorrectly; or does not supply an answer within 3 s, put that card in the pile of unknown facts.
6	Count the number of cards in the known-facts pile to determine the baseline or progress-monitoring score.

Studies have shown positive learning outcomes using high-probability instructional sequences to increase classroom compliance (Belfiore, Basile, & Lee, 2007), facilitate transitions between academic tasks (Lee, 2006), improve communication and social skills (Davis, Brady, Hamilton, McEvoy, & Williams, 1994; Jung, Sainato, & Davis, 2008), and increase academic productivity (Lee, Belfiore, Scheeler, Hua, & Smith, 2004). Decades of research have demonstrated positive effects of explicit, systematic, intensive instruction (Brophy & Good, 1986; Christenson, Ysseldyke, & Thurlow, 1989; Kroesbergen & Van Luit, 2003; Rosenshine & Stevens, 1986). Although these interventions have not been combined for empirical research, they are both effective and are complementary. The high-probability instructional sequences and explicit, systematic, intensive instruction intervention is a novel approach designed to deliver Tier 3 support to students like Justin who do not demonstrate math fact fluency after receiving typical whole-group and small-group classroom instruction at the Tier 1 and Tier 2 levels.

Gathering Baseline and Progress-Monitoring Data

Comparing intervention effects to baseline data is essential for evaluating student progress. Table 1 shows steps to follow for collecting baseline data. Because this is a Tier 3 intervention, these procedures should be used on a weekly basis to evaluate the impact the intervention is having on student learning (Prewett et al., 2012). The assessment procedure described in Table 1 is designed as a curriculum-based measure (L. S. Fuchs & Deno, 1991). The weekly assessment is easy and quick to administer. The steps provide a standardized procedure for assessing student learning. Each time the assessment is given, it is of equivalent difficulty in representing what the teacher wants the student to be able to do at the end of the intervention (i.e., achieving mastery of all single-digit multiplication facts). If the student is not learning at least four or five new facts each week, instructional decisions should be made to address lack of progress. This may include increasing the number of days

Table 2. Intervention Procedures.

Step	Procedure
1	Arrange the unknown-facts pile: If the following facts are not already in the known pile, put them on the top of the unknown-facts pile in this order: multiples of 0, multiples of 1, multiples of 2, doubles (e.g., 2 × 2, 3 × 3, 4 × 4). Next, place the remaining facts with their reciprocal matches back to back (e.g., 3 × 4 and 4 × 3 should be placed one after the other). If the student had one of the reciprocal facts in the known pile but not the other, place its reciprocal fact under the doubles (or before the reciprocal matches).
2	Set a timer for 10 min. Begin by using high-probability instructional sequences using the known-facts pile that was created during baseline or progress-monitoring data collection: Present two (or more) known facts in a row, allowing students to have quick success before presenting the fact on top of the unknown-facts pile.
3	The first time an unknown fact is introduced, the A-B-C teaching procedure should be used following these steps: • Antecedent: Teacher holds up a fact and reads it aloud, giving the answer (e.g., "4 × 4 = 16"). Then the teacher asks the student to supply the answer (e.g., "What's 4 × 4?"). • Behavior: Student supplies the correct answer. • Consequence: Provide praise. • Pause briefly before starting the next trial. Repeat this step three times in a row with the same unknown fact to provide the necessary practice to support memorization.
4	Repeat the high-probability instructional sequences and A-B-C teaching sequence pattern of presenting known and unknown facts (i.e., present two or more known facts before presenting an unknown fact) throughout the session. Every time a known fact is presented, it should be placed at the bottom of the known pile (or pushed to the side). This allows all or most of the known facts to be presented throughout the session to support maintenance of learned facts.
5	The second time an unknown fact is shown, provide an opportunity for a correct response. If incorrect, use the A-B-C teaching procedure in Step 3.
6	An unknown fact should remain on the top of the unknown pile until it is identified within 3 s when it is presented following a series of known facts.
7	Once an unknown fact is identified within 3 s, move it to a second known-facts pile. Each time you present your two or more known facts, take one or more from the original known-facts pile and one from the second known-facts pile. This creates opportunities for students to have repeated exposure to facts that are new to the known-facts pile throughout the session for maintenance purposes.
8	If at any time the student is unable to respond correctly to a fact from one of the known-facts piles within 3 s, provide error correction by immediately using the A-B-C teaching sequence procedures in Step 3. Then move that fact to the top of the unknown-facts pile.

and length of time the student is receiving the intervention or making changes to the intervention procedures to better meet the needs of the student. According to D. Fuchs and Fuchs (2006), when teachers use curriculum-based measurement to inform their instruction, students perform better.

The only materials needed for baseline and intervention are flash cards. It may be best to create flash cards using blank note cards. Many flash cards available for purchase have pictures, borders, and answers to the fact on the reverse side, which can be distracting to the student. There should be a total of 80 flash cards for the 2-through-9 multiplication facts. It is not necessary to create flash cards for the 0s and 1s because they are all represented in the 2-through-9 facts. When creating the cards, it is important to write all of the facts, including their reciprocals (e.g., 2 × 5 and 5 × 2).

Justin's Baseline Data

At baseline, Justin was able to recall an average of 56 of the 80 single-digit multiplication facts. Baseline data were collected on three separate occasions, with scores ranging from 54 to 57. It is important to have at least three baseline data points to ensure accuracy of preintervention performance. If baseline data are collected only on one occasion, and it happens to be on a day the student is very distracted, emotionally distressed, or physically ill, the data may be skewed.

Intervention Procedures

This intervention is designed as a Tier 3 intervention and requires 1:1 instruction. This instruction can be delivered by the general education teacher, a special education teacher, a paraprofessional, a volunteer, or a university teacher candidate engaging in research or internship experiences. Two or more individuals can implement the intervention as long as fidelity of the intervention is monitored. It is recommended that the intervention occur on a daily basis for approximately 10 min each session to maintain facts that have already been acquired and to develop fluency as new facts are learned (Clarke, Baker, & Chard, 2008; Gersten et al., 2009). Thus, it is best to plan for 5 days of intervention each week, understanding that factors such as student absences, schedule changes, special events, and standardized testing may interfere with intervention sessions. Table 2 provides step-by-step instructions for implementing the intervention. The steps provided can be used as a checklist when monitoring fidelity.

Justin's Intervention Results

Justin's participation in this Tier 3 multiplication facts intervention was very successful. Special education teacher candidates from the university located in close proximity to Justin's school implemented the intervention in a 1:1 setting 4 to 5 days each week for 10 min each session. Although Justin did participate each day in regularly scheduled math lessons targeting a variety of grade-level math skills, he did not receive any other specialized instruction related to learning multiplication facts while receiving this Tier 3 intervention.

After the 1st week of intervention, Justin increased his known facts to 58/80. After Week 2, this increased to 63/80. At the end of Week 3, he knew 71/80 facts. At Week 4, he mastered 74/80 facts. After 5 weeks of intervention, he reached mastery, demonstrating that he knew all 80 facts. Justin maintained fluency of all single-digit multiplication facts 3 weeks following the termination of the intervention. Justin's teacher reported that his success with the intervention positively impacted his performance in the classroom during mathematics instruction with an increased level of motivation and academic success during math lessons.

Conclusion

Teachers will likely find that the high-probability instructional sequences and explicit, systematic, intensive instruction intervention discussed in this article is feasible and effective. It is fairly easy to implement using the procedures detailed in Table 2. The intervention is extremely cost-effective, requiring only the purchase of blank note cards. Scheduling the intervention can be flexible since it requires only 10 min of instruction each day. Thus, teachers can select periods of the day that are most productive for the student and least likely to result in the student's missing essential instruction. This intervention can also be used to teach fluency of addition, subtraction, and division facts as well as a variety of other discrete skills, such as number and letter identification, letter sound identification, shapes identification, coins and coin values identification, mathematical computation skills (Lee et al., 2004), reading fluency (Vostal & Lee, 2011), sight word identification (Burns et al., 2009), decoding skills, and spelling skills.

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Note

1. The vignette provided is an authentic situation and only the name of the student is changed to a pseudonym.

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